Lab #3

The purpose of the lab is to understand surface graphs and their tangent planes.

As we did last week, we will consider the function $F(x, y) = -x^4 + 2x^2 + y^4 - 2y^2 + xy - 1$. We will attempt to find the tangent plane to the graph of F at the point $(1, 1/2, 1/16)$.

1. On a plot of the graph of F, sketch the intersection of the surface with the plane $y = 1/2$ (pick a nice view on the range `x=-1.6..1.6`, `y=-1.6..1.6`). In this part of the lab we will parametrize the tangent to the surface that lies in this direction.

   (a) Fix $y = 1/2$ and write down the single-variable function $g(x) = F(x, 1/2)$. What does this function represent geometrically? Parametrize the tangent line to g at the point $x = 1$. Hint: The line passes through the point $(1, g(1))$.

   (b) Test your computation by using Maple's `plot` command to represent the tangent line and the graph of g on the same plot. (Hint: remember that you can think of the graph of g as a parametrized image by considering $x \mapsto (x, g(x))$. Look on your Maple summary sheet to see how to display two parametrized curves on the same plot.)

   (c) Print out the Maple plot and include it in your lab report.

2. Now fix $x = 1$ and consider the single-variable function $h(y) = F(1, y)$. What does *this* function represent geometrically? Illustrate your conclusion with a sketch on the same plot of the surface from problem 1.

   (a) Parametrize the tangent line to h at the point $y = 1/2$.

   (b) Test your computation by using Maple's `plot` command to represent the tangent line and the graph of h the same plot. Again, print out the Maple plot and include it in your lab report.

3. Put these two ideas together to parametrize the tangent plane to the graph of F at the point $(1, 1/2, 1/16)$, by doing the following.

   (a) Determine the tangent vectors to the surface graph in each coordinate direction.

   (b) Describe how to use these vectors to parametrize the tangent plane.

   (c) Test your idea by using Maple's `plot3d` command to simultaneously display the graph of F and the tangent plane at $(1, 1/2, 1/16)$. Indicate as best you can the point of tangency.

4. Generalize this procedure and write a letter to a friend in Calc II that describes how to find the tangent plane to a graph at an arbitrary point $(x_0, y_0, F(x_0, y_0))$. Be sure to describe the geometry of the situation and sketch plenty of diagrams to accompany the prose.