Mad About Mirrors

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Abstract

During my project for the 1995 Summer Institute, I created a video. The video, titled *Mad About Mirrors*, illustrates concepts of reflective symmetry. It is approximately six minutes long.

1 Introduction

1.1 *KaleidoTile*

KaleidoTile originated as The Geometry Center's Triangle Tiling Exhibit at the Science Museum of Minnesota. Jeff Weeks ported it to the MacIntosh Environment. *KaleidoTile* is rich in mathematical concepts, especially those relating to symmetry. However, the program doesn't teach these concepts. Thus, we decided to develop a curriculum to present these concepts in a way that makes *KaleidoTile* immediately useful as a learning tool.

1.2 Teacher Team

A group of middle- and high-school teachers teamed up with undergraduates from The Geometry Center's 1995 Summer Institute. The mission of the resulting *Kaleidotile* Teacher Team was to develop the curriculum to support *Kaleidotile*. The group divided its members between two tasks: the creation of lessons and of a video.

1.3 Purpose of the Video

The purpose of the video, titled *Mad About Mirrors* is to illustrate several related mathematical concepts: simple reflective symmetry; reflective tessellations and tilings; and the workings of planar and polyhedral kaleidoscopes. To inspire further exploration into these subjects, the video does not exhaust all possibilities in any of them. For instance, only selected planar kaleidoscopes are demonstrated.

1.4 Intended Audience

Mad About Mirrors is part of the *KaleidoTile* teaching curriculum, and is meant to be used in that context. Ideas only hinted at in the video are explained in detail in the accompanying lessons. Still, an isolated viewing of the video presents many ideas that an enthusiastic math student might pursue independently.

The curriculum is aimed at students in the sixth through ninth grades. Though older students can assuredly learn from it, they may find the method of presentation in the video to be too juvenile.

2 The Content of the Video

2.1 Characters

The video's main character, Dexter Triangle, is a green scalene triangle with a maroon-colored eye and a blue smile. Dexter offers colorful commentary with a drawl.

The other voice that appears is the narrator's, who explains the action. The narrator also sets up Dexter for some funny observations.

2.2 Plot

The video is broken into several scenes, and a full script may be obtained at **http://www.geom.umn.edu/~teach95/kt95/mad.html** on the World Wide Web. Here is a brief synopsis of each scene.

2.2.1 Title

The video's title, *Mad About Mirrors*, flies around reflective spheres and eventually centers itself so that the viewer can read the title.

2.2.2 Triangle and Mirror Arrive

Dexter Triangle spins gleefully into view and gets introduced. A few moments later, a mirror arrives so that Dexter's reflection is visible in it. Also visible, of course, is the reflection of the tabletop that both Dexter and the mirror are resting on.

2.2.3 Mirror Tilts

The mirror tilts back and forth, as does the reflection within it.

2.2.4 Second Mirror Arrives

Another mirror comes into view, and lands opposite the first one. Dexter sits between them.

2.2.5 Tilting Corridor

The camera moves into the space between the mirrors, and looks into one of them. The image is of an "infinite" corridor, whose length is in fact limited to about thirty reflections. The discontinuity is not noticeable, though, because each successive reflection is smaller than the previous one, and the thirtieth reflection is small enough to be practically invisible.

The viewer might be reminded of the "barbershop" corridor that occurs when one sits between two mirrors. Indeed, the phenomenon is the same. Next, one of the mirrors tilts, causing the the corridor to do so as well.

2.2.6 Dihedral Symmetry

The camera pulls back, and the mirrors rearrange themselves such that they are edge to edge with a $90°$ angle between them. A label of "$90°$" appears and holds position for a few moments. Then, the label disappears, the camera moves in, and the viewer sees the original Dexter and three copies of him in the mirror.

After pausing, the camera retreats again, and the sequence is repeated for $45°$ and $60°$. Each time, a label for the angle appears, and the proper number of reflections are visible.

2.2.7 Third Mirror Arrives

A third mirror appears, rather dramatically. It lands so that the three mirrors form a $60°$–$60°$–$60°$ triangle. This constitutes a kaleidoscope. Dexter sits in the middle of this kaleidoscope.

2.2.8 The $60°$–$60°$–$60°$ Kaleidoscope

The camera moves into the kaleidoscope, and pans $360°$. The reflectivity is 99% for each of the three mirrors during this scene, so images can be seen receding into the distance, illustrating the nature of a reflective tessellation.

2.2.9 The $45°$–$45°$–$90°$ Kaleidoscope

The camera retreats, and the mirrors rearrange themselves into a $45°$–$45°$–$90°$ triangle. Numbers appear at each corner to label the angles. The camera enters the new kaleidoscope and pans $360°$.

2.2.10 Cubic Kaleidoscope

The mirrors rearrange themselves so that they are no longer perpendicular to the plane. When the rearrangement is complete, the mirrors lie along the planes of symmetry of a cube, and the new kaleidoscope extends into the plane. The kaleidoscope rotates to show some of the many different shapes with cubic symmetry. Dexter's images are very helpful here, helping the viewer grasp that a three-dimensional image has been created by the reflection of a two-dimensional object.

2.2.11 Tetrahedral Kaleidoscope

The picture fades to a view of a tetrahedral kaleidoscope, and this change is explained by the narrator, with questions thrown in by Dexter. The tetrahedral kaleidoscope is explored in the same way as the cubic one.

2.2.12 Tiling of Space

A fourth mirror arrives, and caps the other three to form a non-regular tetrahedron of mirrors. The camera sits inside this kaleidoscope, which has one corner poking through the plane so that Dexter is visible on the faces of a cube. The camera pans around, showing a view that goes on "forever" in every direction.

2.2.13 Credits

Five pages of credits then appear, acknowledging technical assistance, helpful feedback, the software used, KaleidoTile, the Geometry Center, and the National Science Foundation.

3 Software and Hardware

3.1 The SOFTIMAGE Creative Environment

SOFTIMAGE proved to be an excellent animating tool. Its tutorial was very helpful, and I was able to begin work on a draft only three days after picking up a manual. Besides having all of the necessary animation and rendering tools, it can render scenes in the background on four of the Geometry Center's computers, freeing their consoles for other work. This is especially useful for the multi-mirror scenes, for which a high level of recursion and a great deal of computational time was needed (up to three days for twenty seconds of film).

For the dihedral scenes, each mirror must have a reflectivity of at least 80%. For the scenes with many reflections, however, the reflectivity must be set at at least 97%. The 60°–60°–60° kaleidoscope scene required that each mirror's reflectivity be set at 99%. To avoid continuity flaws, it is necessary to be extremely careful with material animations. Such a flaw is present in the video, but is not glaring. It should, however, be fixed in any suceeding release of the video.

3.2 Heave

Heave is a program, written at the Geometry Center, which transfers individual image files to specific frames of videotape. I wrote a recording script, with the help of Stuart Levy, to place each frame of the video in the correct order.

This is naturally preferable to transferring each frame individually! Still, Heave doesn't always get perfect timing, and skips images that it can't record with proper timing. Thus, it was necessary to transfer certain frames one at a time (luckily not many), which was time-consuming.

3.3 Silicon Graphics

Computation was performed on an Iris Crimson and Iris Indigos. They run under the IRIX IV or IRIX V operating system.

4 Future Work and Improvements

Though the video is ready for beta-testing, there are several improvements that can be made if someone wishes to continue work on it.

4.1 Continuity

The animation of the reflectivity must be edited in some of the middle scenes to prevent continuity flaws. This can be done using SOFTIMAGE's function curve editor. However, it will require completer re-rendering of the scenes involved.

4.2 Dexter

In the beginning of the film, Dexter is introduced as "Dexter Triangle of Flatland". However, Dexter makes comments about the action that only a three-dimensional observer might make. Though A. Square, the hero of *Flatland*, attained three-dimensional insight by the end of *Flatland*, this is not common knowledge, and so Dexter's comments may provide a source of confusion.

4.3 Borders

The borders on the mirrors are currently not as thick as the mirrors themselves. They should be resized. This requires re-rendering any scene in which the edges of the mirrors appear.

4.4 Lighting

Currently, two lights illuminate the action. I think that adding at least one more would make the video more visually appealing.

4.5 Arrival of the Fourth Mirror

The narration explains what is happening here, but there is probably a clearer way to introduce the fourth mirror. At present, it's hard to see what's going on, though the narration is clear enough.

4.6 Tilting Mirrors

In the scenes in which mirros tilt, instant replays from other angles may be helpful, since it's easier to visualize things that you view from different positions.

4.7 Square Kaleidoscope

Perhaps a square kaleidoscope should be added to the sequence of planar kaleidoscopes. It might also be nice to add an icosascope to the sequence of polyhedral kaleidoscopes

4.8 Motion of Polyhedral Kaleidoscopes

It should be easier to see that the kaleidoscope moves partially into the plane and rotates. As the scene now stands, the only view is from the inside of the kaleidoscope. An instant-replay, from a different angle, may be helpful. Also, in the fade from the cubic kaleidoscope to the cubic kaleidoscope with clipped edges, things should be changed so that the fade-out image and fade-in image are identical execpt for the clipping of the protruding edges. Currently, the kaleidoscope's orientation is different in the fade-out image than the fade-in image.

4.9 Audio

The Geometry Center lacks the proper equipment to record two voices and music. As a result, there is an unpleasant echoic quality to the voices of Dexter and the Narrator. Extraneous noises are also audible.

5 Conclusions

Mad About Mirrors fits well with the *KaleidoTile* curriculum. I have written several pages for the World Wide Web which offer still and animated segments of the video. These pages tie into the *KaleidoTile* curriculum's pages. I am pleased with the final video, and consider my summer to have been well spent.

6 Acknowledgments

Technical guru-ship was provided by Stuart Levy and Lori Thomson. They taught me about various pieces of software and hardware, eased me into better usage of UNIX, and generally made *Mad About Mirrors* possible.

Chaim Goodman-Strauss provided Dexter's voice and personality, and also offered helpful advice about the video's content. Rolf Hanson composed and played the music. Members of the Early Alert Program, the *KaleidoTile* Teacher Team, and the 1995 Summer Institute provided useful feedback.

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