Pisces: A Platform for Implicit Surfaces and Curves and the Exploration of Singularities

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Pisces was developed at the Geometry Center, at the University of Minnesota. The Geometry Center is funded through the National Science Foundation.

Pisces was produced through the efforts of many people: Frederick J. Wicklin, Mark Phillips, Davide Cervone, Daniel Krech, Chia-Hsing "Josh" Nien, and Nicolas Vera.

The following students also contributed to Pisces during the Geometry Center's 1995 Summer Experience for Undergraduates: Ron Dror, John Golden, Brian Johnson, Erik Streed.

Chapter 1

Introduction

This manual is available on-line at the URL

`http://www.geom.umn.edu/locate/pisces/`

The on-line version may be more current, and may contain features not contained in the printed version. For example an the on-line version contains an interactive interface to Pisces via the World-Wide Web.

1.1 What is Pisces?

Pisces is a Platform for Implicit Surfaces and Curves and the Exploration of Singularities. Each term in the acronym has meaning, but, in a nutshell, Pisces is a project that attempts to supply the mathematical and scientific communities with numerical tools to compute and visualize curves and surfaces that are defined *implicitly*. For a simple example, note that the equation to *x^2+y^2=1* is satisfied by the set of points on in the plane that lie on the unit circle, so that the equation implicitly defines the unit circle. One product of the Pisces project is a software package that provides a variety of state-of-the-art algorithms for computing implicitly-defined curves and surfaces. This software is under development.

- Overview: A Pisces Slideshow

- The Main Idea: Level Sets
- Goals of the Pisces Project
- Potential Users
- Current Status of the Pisces Software
- The Pisces Development Team

1.2 Goals of the Pisces Project

The goal of this project is to create software that contains

- A set of known continuation algorithms for computing implicitly-defined curves and surfaces. This will include both piecewise-linear methods and also predictor-corrector methods;
- A command language for batch processing and high level programming;
- A graphical interface for choosing and controlling algorithms;
- New algorithms for computing the structure of singular curves and surfaces.

In addition to being a useful tool to researchers and educators who need to compute implicitly-defined surfaces, the Pisces program will serve as an environment that can be used to study the way that algorithms fail and to develop algorithms that perform better.

1.3 Potential Users

Pisces will be potentially useful to:

Researchers in

- Algebraic Geometry (real and complex algebraic varieties)
- Dynamical Systems (bifurcation surfaces, invariant manifolds)
- Differential Geometry (surfaces, curves, singularities)
- Differential Equations (DEs on manifolds, generation of finite element meshes)
- Mathematical Biology (nonlinear models in physiology, epidemiology, neurology)
- Mathematical Physics (equipotentials, caustics)

Educators in

- Algebraic Geometry (real algebraic varieties)
- Differential Geometry (surfaces, curves, envelopes)
- Dynamical Systems (level sets, elementary bifurcations, normal forms)

Industrial Scientists

- Computer-Aided Geometric Design (CAGD)
- Solid Modeling
- Optimization (solution manifolds for families of nonlinear equations)

Students in

- Algebraic Geometry
- Dynamical Systems
- Multivariable Calculus

1.4 Current Status of the Pisces Software

In the fall of 1995 we will begin distributing Pisces to test-sites outside of the University of Minnesota. The documentation you are reading is part of this process. You may run an interactive demo of Pisces before deciding whether to download Pisces.

We are currently studying the behavior of algorithms in the neighborhood of singular curves; we are developing algorithms that detect and locally resolve the geometric structure of singular curves in a neighborhood of the singularities. We do not know of any algorithms that can carry out this task for level sets of generic functions.

We hope to extend this study to provide users with the first general-purpose algorithm that correctly identifies and depicts singular surfaces. We are using ideas from Mike Henderson at IBM's T. J. Watson Research Center to construct an algorithm that computes implicitly-defined (nonsingular) surfaces.

In the future we will continue our efforts to provide users with high-level mechanisms to compute and graphically display level curves and surfaces by means of a Pisces command language. We will concentrate on communication between algorithms, so that Pisces becomes an extensible platform for computing and visualizing implicitly-defined curves and surfaces.

1.5 The Pisces Development Team

Pisces was developed at the Geometry Center, at the University of Minnesota. The Geometry Center is funded through the National Science Foundation.

Below each name is a brief (and noninclusive!) list of some of the portions of Pisces that that person worked on.

Frederick J. Wicklin (**Project Leader**).

Continuation Algorithms (Geisow, Predictor-Corrector, Simple), Derived Models, Numerical Linear Algebra, Printing, Singularities, WebPisces.

Mark Phillips (**Software Engineer**).

Animation, Derived Models, Graphics, Meta-Model, User Interface.

Davide Cervone.

Piecewise Linear Algorithms (Adaptive Grid, Box).

Daniel Krech.

Piecewise Linear Algorithms (Adaptive Grid, Box, Recursive Surface).

Chia-Hsing "Josh" Nien.

Derived Models.

Nicolas Vera.

Derived Models, Numerical Linear Algebra, Predictor-Corrector Algorithm, Singularities, User Interface.

Undergraduates.

Some undergraduate students from the Geometry Center's 1995 Summer Institute were involved in the Pisces Project. They worked under the supervision of Wicklin and Phillips on Animation, Graphics, Predictor-Corrector Algorithm, Printing, Singularities, and WebPisces.

- Ron Dror
- John Golden
- Brian Johnson
- Erik Streed

Chapter 2

Beginning and Ending a Pisces Session

2.1 Starting and Ending a Pisces Session

2.1.1 On a UNIX Workstation

To begin a Pisces session, type
`% pisces`
from a UNIX shell. By default, Pisces will create the Main Panel from which you can control Pisces. To end a Pisces session, type `quit` or `exit` in the Pisces shell, or select `File->Quit` from Pisces' Main Panel.

2.1.2 Command-line Options

Pisces has several command-line options:

`-devel`

read in Tcl files in the current directory.

`-help`

prints out a syntax message and exits.

`-tk`

runs Pisces with the Tk window interface

`-tcl`

runs Pisces without the Tk windows, but with the Tcl shell.

`-version`

prints out the current version of Pisces and exits.