Symmetry

Symmetries create patterns that help us organize our world conceptually. Symmetric patterns occur in nature, and are invented by artists, craftspeople, musicians, choreographers, mathematicians, ...

In mathematics, a precise way to think about this subject is the idea of *a symmetry*. We will talk about *plane symmetries*, those that take place on a flat plane, but the ideas generalize to spatial symmetries, too.

A *plane symmetry* is a way of moving all the points around the plane so that their positions relative to each other remain the same (though their absolute positions may change). For example, *rotation* by 90° about a fixed point is a symmetry.

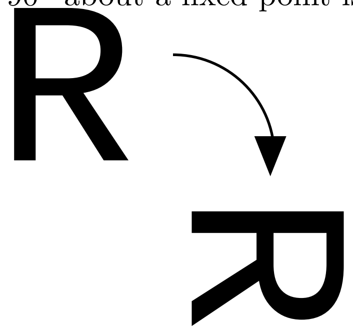

Fig. 1. Rotation of the solid R gives the shaded R.
Where is the center of this rotation?

The distance from the top to the bottom of the solid R is the same as the distance from the top to the bottom of the shaded R; the widths are the same, as are any other measurements you care to make. Symmetries preserve distances, angles, size, and shape.

Another basic type of symmetry is a *reflection*. The reflection of a figure in the plane about a line is where its reflected image would appear to be if you used a mirror placed on the line.

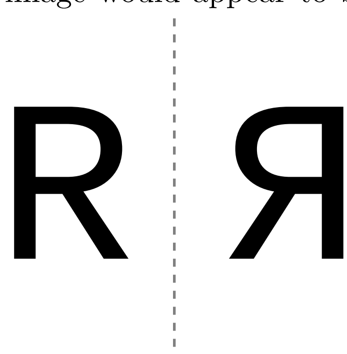

Fig. 2. Reflection of the solid R across the line gives the shaded R.

Another way to make a reflection is to fold the paper and trace the figure onto the other side of the fold.

A third type of symmetry is *translation*. To translate an object means to move it without rotating or reflecting it. You can describe a translation by how far it moves things and in what direction. (See Fig. 3.)

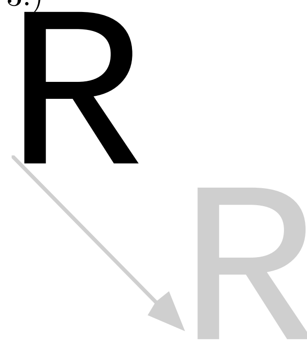

Fig. 3. The solid R has been translated about 3.5 cm southeast.

The fourth (and last) type of symmetry is a *glide reflection*. A glide reflection is a combination of a reflection with a translation along the direction of the mirror line. (See Fig. 4.)

Fig. 4. Glide reflection.

In Figure 4, the solid R has been reflected across the line to give the outlined R (intermediate stage),then the outlined R has been translated in the direction of the same line to give the shaded R. The shaded R is a glide reflection of the solid R.

A figure, picture, or pattern is said to be *symmetric* if there is at least one symmetry that leaves the figure unchanged.

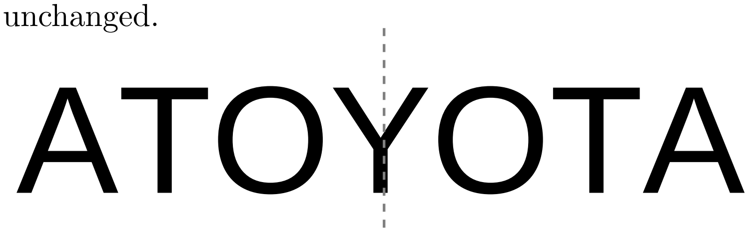

Fig. 5.

For example, the letters above in "ATOYOTA" form a symmetric pattern: if you reflect the entire phrase across the dotted line, the left side becomes the right side and vice versa. The picture doesn't change.

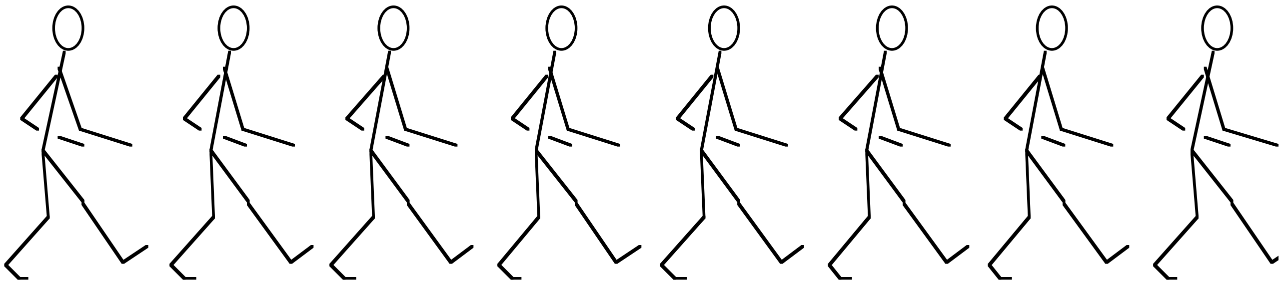

Fig. 6.

If the line of walkers in Fig. 6 is infinite in both directions, then it is a symmetric pattern. You can translate the whole group ahead one person, and the procession will look the same. This pattern has an infinite number of symmetries, since you can translate forward by 1 person, or 2 people, or 3 people, . . ., or backwards by the same amounts, or even by 0 people. That is, there is one symmetry of this pattern for each integer (positive, negative, and zero whole numbers).

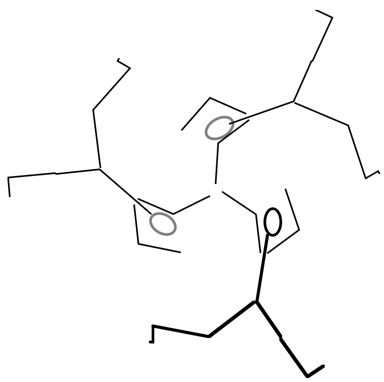

Fig. 7.

The pattern of dancers in Fig. 7 is symmetric because you can rotate the whole picture by 0°, by 120°, or by 240° and the picture will look exactly the same. It is said to have three-fold symmetry, because it has three symmetries, if you include the do-nothing (rotate by 0°) symmetry.